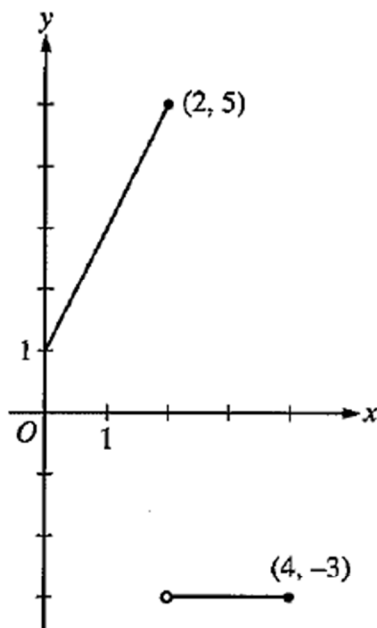


1. If $y = \sin^3 x$, then $\frac{dy}{dx} =$
- (A) $\cos^3 x$ (B) $3\cos^2 x$ (C) $3\sin^2 x$ (D) $-3\sin^2 x \cos x$ (E) $3\sin^2 x \cos x$
2. The position of a particle moving in the xy -plane is given by the parametric equations $x(t) = t^3 - 3t^2$ and $y(t) = 12t - 3t^2$. At which of the following points (x, y) is the particle at rest?
- (A) $(-4, 12)$ (B) $(-3, 6)$ (C) $(-2, 9)$ (D) $(0, 0)$ (E) $(3, 4)$

Graph of f

3. The graph of f is shown above for $0 \leq x \leq 4$. What is the value of $\int_0^4 f(x) dx$?
- (A) -1 (B) 0 (C) 2 (D) 6 (E) 12
4. Which of the following integrals gives the length of the curve $y = \ln x$ from $x = 1$ to $x = 2$?
- (A) $\int_1^2 \sqrt{1 + \frac{1}{x^2}} dx$
- (B) $\int_1^2 \left(1 + \frac{1}{x^2}\right) dx$
- (C) $\int_1^2 \sqrt{1 + e^{2x}} dx$
- (D) $\int_1^2 \sqrt{1 + (\ln x)^2} dx$
- (E) $\int_1^2 (1 + (\ln x)^2) dx$

5. The Maclaurin series for the function f is given by $f(x) = \sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$. What is the value of $f(3)$?

- (A) -3 (B) $-\frac{3}{7}$ (C) $\frac{4}{7}$ (D) $\frac{13}{16}$ (E) 4

6. Using the substitution $u = x^2 - 3$, $\int_{-1}^4 x(x^2 - 3)^5 dx$ is equal to which of the following?

(A) $2 \int_{-2}^{13} u^5 du$

(B) $\int_{-2}^{13} u^5 du$

(C) $\frac{1}{2} \int_{-2}^{13} u^5 du$

(D) $\int_{-1}^4 u^5 du$

(E) $\frac{1}{2} \int_{-1}^4 u^5 du$

7. If $\arcsin x = \ln y$, then $\frac{dy}{dx} =$

(A) $\frac{y}{\sqrt{1-x^2}}$

(B) $\frac{xy}{\sqrt{1-x^2}}$

(C) $\frac{y}{1+x^2}$

(D) $e^{\arcsin x}$

(E) $\frac{e^{\arcsin x}}{1+x^2}$

| | | | | |
|-------------------------|-----|-----|-----|-----|
| t (hours) | 4 | 7 | 12 | 15 |
| $R(t)$ (liters/hour) | 6.5 | 6.2 | 5.9 | 5.6 |

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

- (A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$

II. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$

III. $\sum_{n=1}^{\infty} \frac{n+1}{(n)(n+2)(n+3)}$

- (A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

10. $\int_1^4 t^{-3/2} dt =$

- (A) -1 (B) $-\frac{7}{8}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 1

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x . Which of the following statements is true?

- (A) f is continuous but not differentiable at $x = 2$.
(B) f is differentiable at $x = 2$.
(C) f is not continuous at $x = 2$.
(D) $\lim_{x \rightarrow 2} f(x) \neq 0$
(E) $x = 2$ is a vertical asymptote of the graph of f .

12. The points $(-1, -1)$ and $(1, -5)$ are on the graph of a function $y = f(x)$ that satisfies the differential equation

$\frac{dy}{dx} = x^2 + y$. Which of the following must be true?

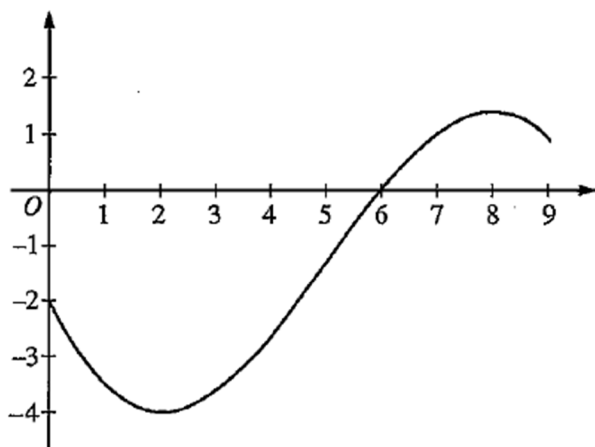
- (A) $(1, -5)$ is a local maximum of f .
(B) $(1, -5)$ is a point of inflection of the graph of f .
(C) $(-1, -1)$ is a local maximum of f .
(D) $(-1, -1)$ is a local minimum of f .
(E) $(-1, -1)$ is a point of inflection of the graph of f .

13. What is the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{3^n}$?

- (A) $2\sqrt{3}$ (B) 3 (C) $\sqrt{3}$ (D) $\frac{\sqrt{3}}{2}$ (E) 0

14. Let k be a positive constant. Which of the following is a logistic differential equation?

- (A) $\frac{dy}{dt} = kt$
 (B) $\frac{dy}{dt} = ky$
 (C) $\frac{dy}{dt} = kt(1-t)$
 (D) $\frac{dy}{dt} = ky(1-t)$
 (E) $\frac{dy}{dt} = ky(1-y)$



Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t) dt$, which of the following is true?

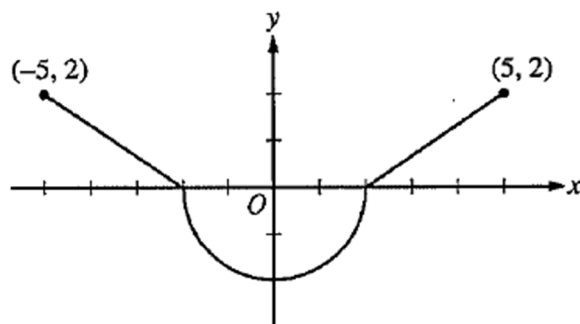
- (A) $h(6) < h'(6) < h''(6)$
 (B) $h(6) < h''(6) < h'(6)$
 (C) $h'(6) < h(6) < h''(6)$
 (D) $h''(6) < h(6) < h'(6)$
 (E) $h''(6) < h'(6) < h(6)$

16. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - y$ with initial condition $f(1) = 3$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

- (A) $-\frac{5}{4}$ (B) 1 (C) $\frac{7}{4}$ (D) 2 (E) $\frac{21}{4}$

17. For $x > 0$, the power series $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots + (-1)^n \frac{x^{2n}}{(2n+1)!} + \dots$ converges to which of the following?

- (A) $\cos x$ (B) $\sin x$ (C) $\frac{\sin x}{x}$ (D) $e^x - e^{x^2}$ (E) $1 + e^x - e^{x^2}$



Graph of f'

18. The graph of f' , the derivative of a function f , consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$

- (A) $2\pi - 2$
 (B) $2\pi - 3$
 (C) $2\pi - 5$
 (D) $6 - 2\pi$
 (E) $4 - 2\pi$

19. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?

- (A) $(0, 0)$ only
 (B) $(\frac{1}{2}, \frac{1}{5})$ only
 (C) $(0, 0)$ and $(-4, 2)$
 (D) $(0, 0)$ and $(4, \frac{2}{3})$
 (E) There are no such points.

20. $\int_0^1 \frac{5x+8}{x^2+3x+2} dx$ is

- (A) $\ln(8)$ (B) $\ln(\frac{27}{2})$ (C) $\ln(18)$ (D) $\ln(288)$ (E) divergent

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

- (A) $y = \frac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \frac{1}{x-5}$ (D) $y = \frac{5x}{1-x}$ (E) $y = \frac{20x^2 - x}{1 + 4x^2}$

22. The power series $\sum_{n=0}^{\infty} a_n (x - 3)^n$ converges at $x = 5$. Which of the following must be true?
- (A) The series diverges at $x = 0$.
(B) The series diverges at $x = 1$.
(C) The series converges at $x = 1$.
(D) The series converges at $x = 2$.
(E) The series converges at $x = 6$.
23. If $P(t)$ is the size of a population at time t , which of the following differential equations describes linear growth in the size of the population?
- (A) $\frac{dP}{dt} = 200$
(B) $\frac{dP}{dt} = 200t$
(C) $\frac{dP}{dt} = 100t^2$
(D) $\frac{dP}{dt} = 200P$
(E) $\frac{dP}{dt} = 100P^2$
24. Let f be a differentiable function such that $\int f(x) \sin x \, dx = -f(x) \cos x + \int 4x^3 \cos x \, dx$. Which of the following could be $f(x)$?
- (A) $\cos x$ (B) $\sin x$ (C) $4x^3$ (D) $-x^4$ (E) x^4
25. $\int_1^{\infty} xe^{-x^2} \, dx$ is
- (A) $-\frac{1}{e}$ (B) $\frac{1}{2e}$ (C) $\frac{1}{e}$ (D) $\frac{2}{e}$ (E) divergent
26. What is the slope of the line tangent to the polar curve $r = 1 + 2 \sin \theta$ at $\theta = 0$?
- (A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

27. For what values of p will both series $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ and $\sum_{n=1}^{\infty} \left(\frac{p}{2}\right)^n$ converge?

(A) $-2 < p < 2$ only

(B) $-\frac{1}{2} < p < \frac{1}{2}$ only

(C) $\frac{1}{2} < p < 2$ only

(D) $p < \frac{1}{2}$ and $p > 2$

(E) There are no such values of p .

28. Let g be a continuously differentiable function with $g(1) = 6$ and $g'(1) = 3$. What is $\lim_{x \rightarrow 1} \frac{\int_1^x g(t) dt}{g(x) - 6}$?

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

(E) The limit does not exist.